THE INFLUENCE OF IRRIGATION ON THE ENERGY BALANCE AND THE CLIMATE NEAR THE GROUND

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ABSTRACT

A theoretical analysis is presented of the influence of irrigation on temperature and humidity of the lower air layers and on the energy balance of the surface. Starting from meteorological data for the dry land (averaged over periods of a few days or longer), the average temperature and moisture profiles in an irrigated area are calculated as functions of the distance downwind from its boundary. The principal simplifying assumption in the analysis is that for each height the eddy diffusivities should have the same values in the irrigated and non-irrigated areas.

The theory is applied to and illustrated by measurements of climatic differences between irrigated and non-irrigated pastures in the Australian Riverina. Experimental results of other investigators are briefly discussed.

The present developments have led to a theoretical estimate, taking advective energy into account, of the potential evaporation rate for irrigated areas of limited extent on the basis of standard meteorological data for the dry land. The influence of advection decreases rapidly with increasing distance downwind. Under summer conditions in the Australian Riverina, it is considerable up to distances of about 1 km.

1. Introduction

In arid and semi-arid regions, irrigation exerts a profound influence on the climate near the ground and on the partition of energy at the surface of the earth. Comparing the situation in an irrigated area with that of the adjacent dry land, it is obvious that the extra water available causes more energy to be consumed in evaporation and less in heating the air and the soil. The temperature of the lowest layers of air is therefore reduced by irrigation, whilst the humidity of these layers is increased. In addition, the lower surface temperature leads to a reduction of the emission of long-wave radiation from the ground. The degree of modification of the energy balance and the climatic elements depends mainly on the irrigation rate, the "dryness" of the dry land, and the rate of advection of warm and dry air into the wet region.

Until recently, little was known quantitatively about these effects, but during the past five to ten years a number of experimental studies of meteorological differences between irrigated and non-irrigated fields have been carried out by Russian workers (*e.g.*, Chudnowskii, 1953, 1954; Dzerdzeevskii, 1952, 1954; Fel'dman, 1953; Gal'tsov, 1953; L'vovich, 1954).

Recently some attention was given to the problem also in the U.S.A. (Halstead and Covey, 1957; Lemon, Glaser, and Satterwhite, 1957; Tanner, 1957).

During the past two years, the author, in co-operation with others, has measured climatic differences between irrigated and dry pastures in the Australian Riverina (a large plain drained by the Murray and Murrumbidgee Rivers). Some results of these experiments are presented here, but a detailed account will be published elsewhere.

The principal aim of this paper is to present a theory which attempts to describe these phenomena quantitatively. Its object is to calculate, on the basis of meteorological data for the dry land, the energy balance and the temperature and humidity regimes of the irrigated area for a given irrigation rate. The present analysis is an extension and modification of that published by Timofeev (1954).

A closely related problem of great practical importance is that of the potential evaporation rate of an irrigated area of limited extent. The present developments have led to a way of estimating this potential rate for irrigated pastures from average standard meteorological data of the dry land.

After explanation of the symbolism in section 2, the theory is expounded in sections 3 to 6. Purely mathematical developments are treated in an appendix. An illustrative example based on the author's experiments is discussed in sections 7 to 9, the potential evaporation rate being treated in section 8. Some further experimental evidence collected from the work of others is discussed in section 10. The paper closes with a brief discussion of the limitations of the theory.

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2. Notation and units [2]

$a = z_1 L\rho_w I/\rho_a c_a K_{H0}$, (C),
$a_1 = z_1 L\rho_w/\rho_a c_a K_{H0}$, (cm^{-1} day C),
A (cal cm^{-2} day^{-1}): surface flux density of sensible heat into the air (positive when away from the surface),
$B = 4z_1\epsilon\sigma T^3{}_{0d}/\rho_a c_a K_{H0}$,
c_a(cal g^{-1}C^{-1}): specific heat of air at constant pressure,
$C = e^\gamma = 1.781$,
d(C): difference between surface temperature and surface dew point for the dry land,
$D = z_1\rho_w I/\rho_a K_{W0}$,
$D_1 = z_1\rho_w/\rho_a K_{W0}$, (cm^{-1} day),
e(mm Hg): pressure of water vapor,
E(cm/day): evaporation rate,
$H_j^{(1)}$: Hankel function of order j,
I(cm/day): irrigation rate,
I_j: modified Bessel function of first kind and order j,
J_j: Bessel function of first kind and order j,
$k = 0.40$: Karman's constant,
K_H(cm^2/day): eddy diffusivity for heat,
K_j: modified Bessel function of second kind and order j,
K_W(cm^2/day): eddy diffusivity for water vapor,
L(cal/g): latent heat of evaporation of water,
m: exponent in power law expression for wind velocity,
n: exponent in power law expression for eddy diffusivities,
p: variable in Laplace transformation,
P(cm/day): precipitation rate,
q: specific humidity,
r: reflection coefficient of surface for short wave radiation,
$\mathbf{r}$(cm): radius vector,
R_a(cal cm^{-2} day^{-1}): intensity of atmospheric (long wave) radiation at the surface,
R_s(cal cm^{-2} day^{-1}): intensity of short wave radiation (from sun and sky) at the surface,
s(C): slope of temperature *versus* saturated specific humidity curve,
S(cal cm^{-2} day^{-1}): surface flux density of heat in the soil (positive when away from the surface),
t(day): time,
T(K): absolute potential temperature,
u(cm/day): horizontal component of wind velocity,
u_*(cm/day): friction velocity,
$\mathbf{v}$(cm/day): vector wind velocity,
x(cm): distance downwind in irrigated area,
X(cm): length of irrigated area in direction of wind,
Y_j: Bessel function of second kind and order j,
z(cm): vertical co-ordinate (positive upwards),
z_1(cm): reference height,
Z(cm): arbitrary large value of z,
$\beta = 2B/(1 + m)$,
$\gamma = 0.5772$: Euler's constant,
ϵ: emissivity of surface for long wave radiation,
$\eta = 1 + z/z_1$: dimensionless height,
$H = 1 + Z/z_1$,
$\theta = T_i - T_d$, (C): temperature difference between irrigated and non-irrigated areas,
ϑ(C): Laplace transform of θ,
μ: fractional cloudiness,
$\xi_H = K_{H0}x/z_1^2u_0$: dimensionless distance downwind,
$\xi_W = K_{W0}x/z_1^2u_0$: dimensionless distance downwind,
ρ_a(g/cm^3): density of air,
ρ_w(g/cm^3): density of liquid water,
$\sigma = 1.176 \times 10^{-7}$ cal cm^{-2} day^{-1}K^{-4}: Stefan-Boltzmann's constant,
$\chi = q_i - q_d$: difference in specific humidity between irrigated and non-irrigated areas.

[2] Subscripts i and d refer to irrigated and dry, respectively; surface values ($z = 0$) are indicated by the subscript 0.

3. The energy balance of dry and irrigated land

The energy balance of the surface of the earth in the dry (subscript d) and irrigated (subscript i) areas is expressed as follows:

$$(1 - r_d)R_{sd} + R_{ad} - \epsilon_d\sigma T_{0d}^4 = S_d + A_d + L\rho_w E_d \quad (1)$$

$$(1 - r_i)R_{si} + R_{ai} - \epsilon_i\sigma T_{0i}^4 = S_i + A_i + L\rho_w E_i. \quad (2)$$

Obviously some of the quantities in (1) and (2) will not differ much when the areas are contiguous and the irrigated area is of limited extent—as, for instance, with dimensions of the order of 10 km or less. Under these restricting conditions, we can write, to a good degree of approximation,

$$R_{sd} = R_{si} = R_s \quad (3)$$

and

$$R_{ad} = R_{ai} = R_a. \quad (4)$$

Regarding (4), we shall see later that the influence of irrigation on air temperature and humidity is restricted to a comparatively shallow layer which contributes only a small fraction of the total atmospheric radiation. In addition, the decrease in temperature and the increase in absolute humidity due to irrigation have opposite effects on the emission of long-wave radiation by this layer.

The average evaporation rate of the dry land will be equal to the average precipitation rate for periods that are not too short—such as the order of a week or more. Hence we have

$$E_d = P_d. \quad (5)$$

Assuming that available water is limiting evaporation (Philip, 1957), we have similarly

$$E_i = P_i + I. \quad (6)$$

In the case of water loss due to drainage, P and I in (5) and (6) must be understood to represent that quantity of precipitation and irrigation water which is evaporated. In the majority of practical cases, the

proportion evaporated will be very close to unity in (5) and not very different from unity (say >0.85) in (6).

From (1) to (6), we then obtain (with $P_d = P_i$)

$$(r_i - r_d)R_s + \sigma(\epsilon_d T_{0d}^4 - \epsilon_i T_{0i}^4) = (S_d - S_i) + (A_d - A_i) - L\rho_w I. \quad (7)$$

The terms $(r_i - r_d)R_s$ and $S_d - S_i$ will often be small in comparison with the other terms in (7) when periods of a week or longer are considered. For the radiation term, this is due to the small difference between r_i and r_d (List, 1951; or Falckenberg and Schnaidt, 1952). The heat fluxes in the soil (S_d and S_i) are comparatively small terms in (1) and (2). Moreover, in comparing both regions, the smaller temperature gradient in the upper soil layers in the wet area is compensated by a greater thermal conductivity of the soil. The author has shown elsewhere (de Vries, 1956) that for the annual cycle these effects almost balance. Chudnowskii (1953) arrived at the same conclusion on the basis of experimental data.

To simplify the notation, we shall therefore reduce (7) further to

$$A_i - A_d = -L\rho_w I - \epsilon\sigma(T_{0d}^4 - T_{0i}^4), \quad (8)$$

where we have also substituted $\epsilon_i = \epsilon_d = \epsilon$. However, if necessary, the neglected terms can be retained without complicating subsequent developments, provided that they are constant.

4. The difference in air temperature

After comparing the energy balances of the two regions, we now turn our attention to air temperature. The essence of the present analysis is that we take the solution which nature presents for the dry land as a starting point and calculate the temperature difference due to irrigation. In addition, we eliminate short-time variations by considering average values over periods of the order of a few days or more.

The equation of heat conduction for the (homogeneous) dry land upwind from the irrigated region is

$$\frac{\partial \rho_a c_a T_d(z,t)}{\partial t} = \frac{\partial}{\partial z}\left(\rho_a c_a K_{Hd}\frac{\partial T_d(z,t)}{\partial z}\right). \quad (9)$$

For the irrigated area, we have

$$\frac{\partial \rho_a c_a T_i(\mathbf{r},t)}{\partial t} = \nabla[\rho_a c_a K_{Hi}\nabla T_i(\mathbf{r},t)] - \nabla[\rho_a c_a \mathbf{v}(T_i - T_1)]. \quad (10)$$

The second term on the right hand side of (10) arises from advective heat transfer; T_1 is an arbitrary reference temperature. Using the continuity equation

$$\frac{\partial \rho_a}{\partial t} = -\nabla(\rho_a \mathbf{v})$$

and neglecting variations of $\rho_a c_a$, equation (10) reduces to

$$\partial T_i/\partial t = \nabla(K_{Hi}\nabla T_i) - (\mathbf{v}\nabla T_i).$$

Now introducing the temperature difference $\theta(x,z,t) = T_i - T_d$, and assuming $K_{Hi} = K_{Hd} = K_H$, we obtain

$$\partial\theta/\partial t = \nabla(K_H\nabla\theta) - (\mathbf{v}\nabla T_i). \quad (11)$$

The assumption $K_{Hd} = K_{Hi}$ will hold true to a good degree of approximation in the lowest air layers where forced convection dominates free convection (provided that the aerodynamic properties of the dry and irrigated surfaces are about the same) and also at heights which are sufficiently great for the influence of irrigation on the temperature profile to be small, so that buoyancy effects will be approximately the same in both cases. There will be an intermediate region where K_{Hd} is systematically greater than K_{Hi}, the depth of which will depend on wind speed and heat flux. However, the assumption $K_{Hd} = K_{Hi}$ is essential for making the problem amenable to analysis; its influence will be discussed further below.

Observation has shown that $\partial\theta/\partial t$ is small for an irrigation rate that is constant in time. In other words, although both T_d and T_i change with time, their difference depends mainly on the irrigation rate.

Neglecting diffusion downwind and across wind, we finally obtain for a horizontal-wind movement in the x-direction the following differential equation for θ:

$$u\frac{\partial\theta}{\partial x} = \frac{\partial}{\partial z}\left(K_H\frac{\partial\theta}{\partial z}\right). \quad (12)$$

From (8), we have the boundary condition

$$-\rho_a c_a\left(K_H\frac{\partial\theta}{\partial z}\right)_{z=0} = -L\rho_w I - 4\epsilon\sigma T_{0d}^3\theta_0. \quad (13)$$

Furthermore,

$$\theta(x,z) = 0, \quad \text{for} \quad z = \infty. \quad (14)$$

The problem has now been reduced to that of heat conduction in a semi-infinite medium with prescribed heat flux at the surface and radiation into a medium at zero temperature. The only complication is that u and K_H are functions of z. A proper choice of these functions must be made so that on the one hand they represent actual conditions to a sufficient degree of approximation and on the other hand lead to manageable solutions. This problem is discussed in the appendix.

Here we present the solution for the case where

$$u = u_0(1 + z/z_1)^m, \quad (15)$$

and

$$K_H = K_{H0}(1 + z/z_1). \quad (16)$$

Here, z_1 is a properly chosen reference height which can be, but need not be, the roughness height of the surface. The influence of the infinite increase of K with height will be considered later.

For u, it would have been preferable to select a logarithmic profile or a profile proportional to z^m. However, both cases lead to great mathematical difficulties. A solution in terms of tabulated functions can be derived for $u \sim z^m$, $K \sim z^{1-m}$, and $(K\partial\theta/\partial z)_{z=0}$ = constant (Timofeev, 1954). However, the latter condition is at variance with equation (13).

The present profile for u only differs markedly from that of a normal power law for z-values of the order of z_1 or less. (In the example discussed below, z_1 is less than 1 cm.)

The solution of (12) to (16), as derived in the appendix, is

$$\theta = \frac{4a}{(1+m)\pi}\int_0^\infty \frac{[vf_1(v,v')+\beta f_2(v,v')]}{v^3f_3(v)+2\beta v^2f_4(v)+\beta^2vf_5(v)} \times\{1-\exp[-\tfrac{1}{4}(1+m)^2\xi_H v^2]\}dv, \quad (17)$$

with

$$\begin{aligned}
\alpha &= z_1L\rho_wI/\rho_ac_aK_{H0},\\
\beta &= 8z_1\epsilon\sigma T_{0d}^3/\rho_ac_aK_{H0}(1+m),\\
\xi_H &= K_{H0}x/z_1^2u_0,\\
v' &= v(1+z/z_1)^{(1+m)/2},\\
f_1(v,v') &= J_0(v')Y_1(v) - Y_0(v')J_1(v),\\
f_2(v,v') &= J_0(v')Y_0(v) - Y_0(v')J_0(v),\\
f_3(v) &= J_1^2(v) + Y_1^2(v),\\
f_4(v) &= J_0(v)J_1(v) + Y_0(v)Y_1(v),\\
f_5(v) &= J_0^2(v) + Y_0^2(v).
\end{aligned}$$

Calculations of θ from (17) are laborious. However, for many β- and ξ_H-values that are of practical interest, θ can be found from simpler asymptotic expressions given in the appendix (equations A14 and A15).

5. The difference in humidity

By an argument similar to that of the previous section and under similar restricting conditions, the following differential equation for the difference in specific humidity, $\chi = q_i - q_d$, of the air in both areas is found:

$$u\frac{\partial\chi}{\partial x} = \frac{\partial}{\partial z}\left(K_W\frac{\partial\chi}{\partial z}\right). \quad (18)$$

The only difference between (18) and (12) is the occurrence of the diffusion coefficient for water vapor, K_W, instead of K_H.

The boundary conditions, analogous to (13) and (14), are

$$-\rho_a\left(K_W\frac{\partial\chi}{\partial z}\right)_{z=0} = \rho_wI, \quad (19)$$

and

$$\chi(x,z) = 0, \quad \text{for} \quad z = \infty. \quad (20)$$

The solution with u, given by (15) and

$$K_W = K_{W0}(1+z/z_1), \quad (21)$$

is

$$\chi = -\frac{4D}{(1+m)\pi}\int_0^\infty \frac{J_0(v')Y_1(v)-J_1(v)Y_0(v')}{v^2[J_1^2(v)+Y_1^2(v)]} \times\{1-\exp[-\tfrac{1}{4}(1+m)^2\xi_W v^2)\}dv, \quad (22)$$

with $D = z_1\rho_wI/\rho_aK_{W0}$, $\xi_W = K_{W0}x/z_1^2u_0$. An asymptotic expression for large ξ_W is given in the appendix (equation A16). Values of χ for $z = 0$ can be found in Carslaw-Jaeger (1948).

Note on Timofeev's analysis.—If we multiply equation (18) by L/c_a and add it to (12), putting $K_H = K_W$, we obtain a similar differential equation for $\theta + L\chi/c_a$—*i.e.*, the difference in equivalent potential temperature between the dry and irrigated areas. This equation was given by Timofeev (1954) without derivation and without stating the simplifying assumptions on which it is based. Timofeev applies this equation to the case where

$$K_{Hi} = K_{Wi} = \alpha K_{Hd} = \alpha K_{Wd}, \quad (23)$$

where α is a constant smaller than 1. This means that he also neglects a term $(1-\alpha)\partial(T_d + Lq_d/c_a)/\partial t$ in his differential equation. This term will only be small when α lies close to 1. Moreover, from a physical viewpoint, the present author considers the assumption (23) with constant $\alpha < 1$ not to be an improvement of the assumption $\alpha = 1$ except when the aerodynamic properties of the dry and irrigated surfaces are very different. In that case, there are further difficulties due to the fact that in a transition region the vertical component of $\mathbf{v}$ must be appreciably different from zero.

In the boundary condition for $\partial(\theta + L\chi/c_a)/\partial z$ following from (13) and (14), the terms with I cancel out. Timofeev further assumes that at the surface $\partial(\theta + L\chi/c_a)/\partial z$ is constant—*i.e.*, independent of x. He shows how this constant depends on the difference in net radiation between the dry and irrigated areas and on the soil heat flux density; the latter quantities must be determined experimentally. Timofeev then gives solutions for $u \sim z^m$, $K \sim z^{1-m}$.

Hence, the principal differences between Timofeev's analysis and the present one are: (a) in our treatment, the difference in net radiation is related to the difference in surface temperature between irrigated and dry land; (b) the temperature and moisture profiles are solved for separately and are related to the irrigation rate, whereas Timofeev eliminates I and finds a solution for the equivalent temperature only.

TABLE 1. Average meteorological data for the area (period 8 to 22 December 1957).

Solar radiation (R_s)	7.8×10^2 cal cm^{-2} day^{-1}
Reflection coefficient (r)	0.23
Fractional cloudiness (μ)	0.3
Horizontal wind velocity (u) at 200 cm	
Average	2.5×10^7 cm/day
Diurnal amplitude	1.1×10^7 cm/day
Irrigation rate (I)	0.35 cm/day
Precipitation rate (P)	0.03 cm/day

TABLE 2. Averages and average diurnal amplitudes of meteorological data for the various stations (period 8 to 22 December 1957).

	Station number			
	1	2	3	4
Air temperature, 125 cm (C)				
Average	22.1	20.7	20.0	19.7
Amplitude	8.9	8.7	8.5	8.8
Soil temperature (C)				
Average, −5 cm	30.8		18.5	
Amplitude, −5 cm	8.4		2.5	
Average, −30 cm	27.7		17.3	
Amplitude, −30 cm	1.7		0.4	
Relative humidity, 125 cm				
Average	0.50	0.59	0.61	0.63
Amplitude	0.32	0.32	0.30	0.31
Dew point, 125 cm (C)				
Average	9.4	11.1	11.2	11.3
Precipitation (cm)	0.48	0.55	0.33	0.20

6. Solutions for more complicated cases

In section 3, it was assumed that certain terms in (7) were negligible and, as a consequence, a in (17) is a constant proportional to I. When these terms have to be retained, the solution (17) remains valid if they are constant, a being of the form $c_1 I + c_2$ in this case.

The solutions become more complex when I or other terms in the expression for $A_i - A_d$ are functions of x. When the Laplace transforms of these functions are known, the same procedure as set out in the appendix can be applied but more complicated expressions for θ and χ will be found.

A simple case of practical importance is that of an irrigated area with constant irrigation rate and length X in the wind direction, where we are interested in the behavior for $x > X$—*i.e.*, to the leeward of the irrigated area. For $0 \leq x \leq X$, the previous solutions hold. Writing the solution for θ as

$$\theta = F(x,z), \quad \text{for} \quad 0 \leq x \leq X, \tag{24}$$

we have

$$\theta = F(x,z) - F(x - X,z), \quad \text{for} \quad x > X, \tag{25}$$

whilst a similar solution holds for χ.

The case where u and K are given by (15) and (16) up to a certain height, Z, whilst they are constant above that height, is treated in section A3 of the appendix, whilst potential evaporation is dealt with in section A4.

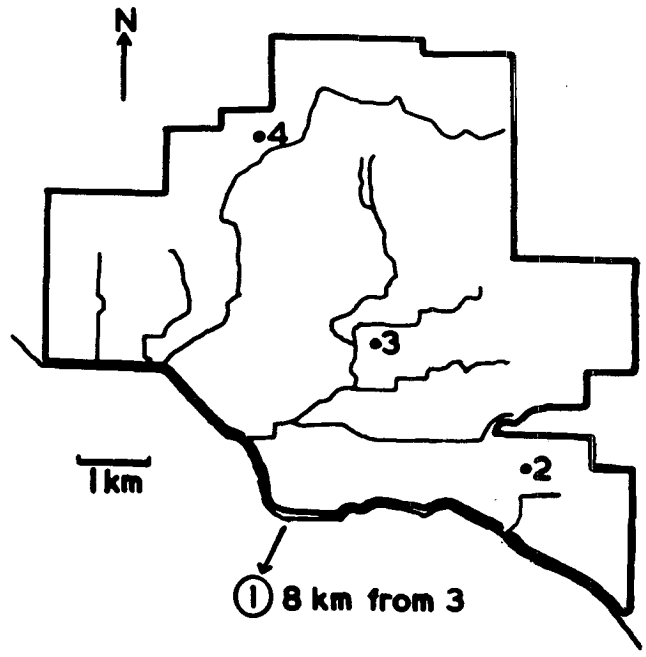

FIG. 1. Plan of the irrigated area, showing irrigation channels and positions of stations. Co-ordinates of station 3 are 36.4 S, 144.8 E.

7. An illustrative example

Experimental lay-out.—Since November 1956, continuous meteorological measurements have been made in the Nanneella irrigation district near Rochester (Victoria). This is a dairying district consisting almost entirely of irrigated pastures. Fig. 1 shows a plan of the intensively irrigated area which is surrounded by dry land apart from some patchy irrigation to the east and north.

Three meteorological stations were established in the area (numbers 2 to 4 in fig. 1). A dryland station (number 1) is situated at a distance of about 6 km to the southwest of the irrigated area. This station is on a dryland pasture which has a sparse vegetation of native grasses. The other stations are on irrigated pastures, carrying a dense cover of mainly perennial ryegrass and irrigation white clover, which are grazed regularly.

Temperatures and relative humidities are recorded by thermohygrographs placed in Stevenson screens at normal height at all stations. In addition, soil temperatures, rainfall, wind direction and velocity are recorded at two stations (1 and 3 or 4). The recordings are supplemented by direct readings of soil and air temperatures, rainfall, cloudiness, wind velocity and direction once a week at all stations but more frequently at some of them (daily at 4). A detailed account of the instrumentation, experimental routine, and method of compiling the data will be published elsewhere.

Results for December 1957.—Some typical results,

averaged for the period 8 to 22 December 1957 are shown in tables 1 and 2. The weather during this period was mostly clear and dry (apart from some showers on 9 December), and winds were predominantly from southerly directions. A fifteen-day period was chosen because this is the interval between successive irrigations.

The energy balance.—We shall now proceed to analyse the energy balance of the various stations from the data given in tables 1 and 2.

Solar radiation is measured at Deniliquin (about 100 km north of Rochester) by means of a solarimetric thermopile (de Vries, 1958b). Values are corrected for differences of cloudiness between Rochester and Deniliquin on the basis of a linear regression of daily total global radiation and fractional cloudiness (de Vries, 1958b). The albedo of the surface was also measured with the thermopile and was found to be 0.23 on the average both for the dry land and the irrigated pastures. Hence, we find $(1 - r)R_s = 6.0 \times 10^2$ cal cm^{-2} day^{-1}.

Long-wave radiation from the atmosphere had to be calculated from an empirical formula. For humid regions at moderate latitudes, a formula of the type proposed by Penman (1948) for the net long-wave-radiation loss from the surface is known to give reasonable results; *e.g.*,

$$R_{\text{net}} = \sigma T_a^4(0.47 - 0.077e_a^{\frac{1}{2}})(1 - 0.8\mu), \quad (26)$$

where T_a and e_a are air temperature and vapor pressure at screen height. Philip (1957) has already commented on the inadequacy of such a formula under dry conditions where a large temperature difference exists between the surface and screen height. He proposed to modify (26) by substituting T_0 for T_a. However, in doing so, the emissivity of the surface is made to depend on cloudiness. We consider it more logical, therefore, to express atmospheric radiation by a formula that contains only atmospheric quantities. As such, we propose

$$R_a = \sigma T_a^4[(0.53 + 0.077e_a^{\frac{1}{2}})(1 - 0.8\mu) + 0.8\mu], \quad (27)$$

which is equivalent with (26) for $T_0 = T_a$ and $\epsilon = 1$. From this formula, we find $R_a = 7.3 \times 10^2$ cal cm^{-2} day^{-1}.

For the average surface temperature, we find 31.5C by extrapolation from the observed soil temperatures and, with $\epsilon = 1$, we obtain $\epsilon\sigma T_0^4 = 10.1 \times 10^2$ cal cm^{-2} day^{-1}. The net long-wave-radiation loss from the surface thus becomes 2.8×10^2 cal cm^{-2} day^{-1}, whereas (26) would yield 1.6×10^2 cal cm^{-2} day^{-1}.

From the observed soil temperatures, we estimate S at 0.2×10^2 cal cm^{-2} day^{-1} (de Vries, 1958a).

The heat consumed in evaporating precipitation is calculated at 0.2×10^2 cal cm^{-2} day^{-1}. Because of very dry weather prior to 8 December 1957, no evaporation of precipitation of an earlier date need to be taken into account.

Finally, from the energy balance, we find a value of 2.8×10^2 cal cm^{-2} day^{-1} for the heat transferred to the air.

For the irrigated stations, the terms $(1 - r)R_s$, R_a, and S are considered to be the same as for the dry land, as was explained in section 3. The evaporation term is found from the precipitation rate plus the irrigation rate. The latter was computed from figures supplied by the Victoria State Rivers and Water Supply Commission at 0.35 cm per day for the area as a whole.

The surface temperature can now be computed from the energy balance using the assumption of equality of eddy diffusivities for the irrigated and the dry land. From the data for station 1, the heat-transfer coefficient between the surface and screen height (125 cm) is

$$A/(T_0 - T_{125}) = 2.8 \times 10^2/9.4 = 0.30 \times 10^2 \text{ cal cm}^{-2}\text{ day}^{-1}\text{C}^{-1}.$$

Thus we have[3] for station 2, with $E = 583 \times 0.38 = 2.2 \times 10^2$ cal cm^{-2} day^{-1}, the following:

$$6.0 + 7.3 - 1.176 \times 10^{-9}T_{0i}^4 = 2.2 + 0.2 + 0.30(T_{0i} - 293.7),$$

from which we obtain $T_{0i} = 298.8$ K, $\sigma T_{0i}^4 = 9.4 \times 10^2$ cal cm^{-2} day^{-1} and $A = 1.5 \times 10^2$ cal cm^{-2} day^{-1}. Similarly, we obtain for station 3: $T_{0i} = 298.3$ K, $\sigma T_{0i}^4 = 9.3 \times 10^2$ cal cm^{-2} day^{-1}, $A = 1.6 \times 10^2$ cal cm^{-2} day^{-1}; and for station 4: $T_{0i} = 298.1$ K, $\sigma T_{0i}^4 = 9.3 \times 10^2$ cal cm^{-2} day^{-1}, $A = 1.6 \times 10^2$ cal cm^{-2} day^{-1}.

Application of theory.—We are now in a position to apply the theory of sections 4 and 5. In doing so, we must assign suitable values to the parameters z_1, u_0, K_{H0}, and K_{W0}. In order to find values for z_1 and K_{H0}, we have applied the formulas for forced convection to the lower air layers, *viz.*

$$K_H = K_W = K_{H0}(1 + z/z_1) = 0.4\mu_*(z + z_1),$$
$$u = 2.5u_* \ln(1 + z/z_1).$$

Using the observed values of u at $z = 200$ cm and of the heat transfer coefficient between 0 and 125 cm given above we obtain $z_1 = 0.36$ cm, $K_{H0} = K_{W0} = 2.25 \times 10^5$ cm^2/day, and $u_* = 18$ cm per sec which

[3] In using the expression $A = 30(T_0 - T_{125})$ for the irrigated stations, we neglect the influence of advective energy in the layer between 0 and 125 cm. It can be easily verified that for the x-values of stations 2, 3, and 4 this leads to an error in A of the order of 0.1 cal cm^{-2} day^{-1} which is negligible.

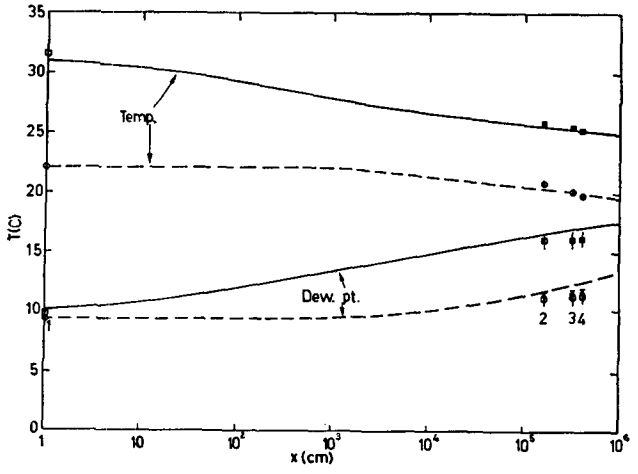

FIG. 2. Calculated temperatures and dew points at the surface (solid curves) and at 125-cm height (broken curves) in relation to distance downwind in the irrigated area for the period 8 to 22 December 1957. Measured values for 125-cm height (screen) are shown by circles, surface values calculated from the energy and water balance by squares. Weighted averages of dew point are shown by crosses. Numbers 1 to 4 refer to the stations; the values for station 1 ($x < 0$) are shown on the axis of ordinates. Vertical lines indicate standard experimental errors in the differences between an irrigated station and the dry station.

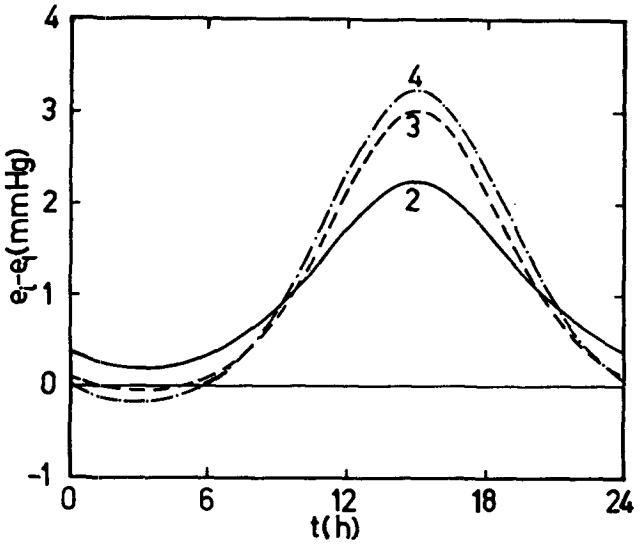

FIG. 3. Diurnal variation of the difference in vapor pressure of an irrigated station, e_i (i = 2, 3, 4, shown on the curves), and the dry station, e_1.

are acceptable values. With $m = 0.125$, we then find $u_0 = 112 \times 10^5$ cm per day and $\xi_H = \xi_W = 0.155x$. These values lead to $a = 1.15$ C, $B = 0.075$, $\beta = 0.133$, and $D = 0.475 \times 10^{-3}$.

Results of calculations from the theory of sections 4, 5 and A2, A3 of the appendix for $z = 0$ cm and $z = 125$ cm are presented in fig. 2. Dew points have been plotted instead of χ-values in order to show temperature and humidity effects on a common scale of ordinates.

In section A3, asymptotic solutions are given for the case where K and u increase linearly with z up to a certain height Z and remain constant for greater heights. In the present example, we have taken $Z = 10^4$ cm (Lettau, 1952). A comparison of the solutions for bounded K and u with that for unbounded K and u shows that there is no difference between the two for $x \leq 10^5$ cm and $z \leq 125$ cm and only a small difference for $x = 10^6$ cm and $z = 125$ cm. In the latter case, the values calculated for bounded K and u have been plotted in fig. 2. Differences between the two solutions are appreciable at $x \geq 10^7$ cm, but for these large x-values the irrigated area is no longer of "limited extent" in the sense in which this term has been used here.

Experimental values of air temperatures at 125 cm height and of surface temperatures calculated from the energy balance are also shown in fig. 2. Average ξ-values were determined by averaging x/u for the period under consideration. To this end, average wind directions were read from the recordings (in multiples of 22.5 deg from north) for each six-hour period, the corresponding distances from the edge of the irrigated area were measured in fig. 1, and these were divided by the average wind velocity for the period.

The standard error in the average temperatures is 0.2C. In fig. 2, the dry-land values have been used as a datum. The standard error in the differences between an irrigated station and the dry-land station is therefore $2^{\frac{1}{2}} \times 0.2 = 0.3$C. This error is shown by vertical lines on either side of the experimental points for stations 2, 3, and 4. The agreement between theoretical and observed values is satisfactory.

Average dew points at 0 and 125 cm are also plotted in fig. 2. The dew points at the surface were obtained from those at 125 cm, the evaporation rates, and the mass transfer coefficient[4] between 0 and 125 cm. The latter is found by multiplying the heat transfer coefficient with 3.42×10^{-3} C/mm Hg.

The standard error in the average relative humidities is 0.01. In combination with the standard error in temperature, this leads to a standard error of 0.5C in the dew points and of 0.7C in the dew-point differences between an irrigated and the dry-land station. The latter value is shown by a vertical line on either side of the experimental points for stations 2, 3, and 4. The systematic differences between calculated and observed values are thought to be connected with the diurnal variation of χ, as will be explained below.

Diurnal variation.—It can be easily verified by taking time averages of equations (12) to (14) over a period of one day that they do apply for diurnal averages of the parameters provided that θ is independent of time. From table 2, it can be seen that the diurnal amplitude of the air temperature at 125 cm is

[4] The influence of advection in the layer of 0 to 125 cm on the results is negligible (*cf.* footnote 3).

approximately the same for all stations. Hence, the approximation of constant θ appears to be reasonable.

The situation is different for air humidity. In fig. 3, the differences in vapor pressure between stations 2, 3, 4 and station 1 are plotted as a function of time; they show a pronounced diurnal variation. For simplicity, vapor pressures were calculated on the assumption that both temperature and relative humidity have a sinusoidal diurnal variation, the two sine curves being in anti-phase. A comparison with values calculated directly from the thermo-hygrograph recordings showed that this is a permissible simplification.[5]

The analysis of section 5 can still be applied if we assume that the diurnal variation of K and u can be expressed as follows:

$$K_W = K_0(1 + z/z_1)f(t), \tag{28}$$

$$u = u_0(1 + z/z_1)f(t), \tag{29}$$

where $f(t)$ is a periodic function with a period of one day ($t = 1$). Taking a time average over one period of the equation (*cf.* 11)

$$\frac{\partial \chi}{\partial t} = \frac{\partial}{\partial z}\left(K_W \frac{\partial \chi}{\partial z}\right) - u\frac{\partial \chi}{\partial x}, \tag{30}$$

we find

$$0 = \frac{\partial}{\partial z}\left(K_W \frac{\partial \chi'}{\partial z}\right) - u\frac{\partial \chi'}{\partial x}, \tag{31}$$

with

$$\chi' = \int_0^1 \chi f(t)dt. \tag{32}$$

Similarly, the boundary condition (19) becomes

$$-\rho_a\left[K_0(1 + z/z_1)\frac{\partial \chi'}{\partial z}\right]_{z=0} = \rho_w I. \tag{33}$$

Hence, the same formulas as derived for χ now apply to χ'. However, it must be kept in mind that the actual variation of K and u with time will differ markedly from that expressed by equations (28) and (29), so that the introduction of χ' can only lead to a rough estimate of the effects caused by the diurnal variation of χ.

Wind-velocity observations suggest the following value of $f(t)$:

$$f(t) = 1 + 0.44 \sin 2\pi t. \tag{34}$$

The maximum of $f(t)$ occurs at about 15h and coincides with the maximum of air temperature and χ. Therefore, the value of χ' is larger than the average of χ. Dewpoints deduced from χ'-values are shown by crosses in fig. 2. They are still lower than the theoretical values, but the differences are well within the experimental and theoretical errors.

[5] The author is indebted to Mr. W. C. Swinbank for suggesting this procedure.

Soil temperatures.—The differences in average soil temperature between stations 1 and 3 are appreciable, *viz.* 10.4C at 30-cm depth, 12.3C at 5-cm depth and (by extrapolation) 12.7C at 0 cm. In comparison, the calculated difference in surface air temperature for the two stations is 6.2C. The discrepancy between these values is ascribed to the effect of the much denser vegetation at the irrigated station. The grass cover not only shades the soil but acts in its lowest layers as a mulch with a low thermal conductivity and low volumetric heat capacity. This "blanketing" effect of a grass cover was also observed and discussed by Peerlkamp (1944). We further refer to papers by de Vries and de Wit (1954) and van Duin (1956) for an analysis of the mulch effect.

Similar effects were observed for stations 2 and 3, but here only occasional readings of soil thermometers were available. Comparing these with the soil temperatures recorded at station 1 at the same time led to the following average differences: for station 2, 14.5C at 5-cm depth and 8.9C at 30-cm depth (averaged from readings on 9 days in the period 8 to 22 December 1957 at approximately 18h); for station 4, 7.1C at 5-cm depth and 8.3C at 30-cm depth (averaged from readings on 14 days at about 9h). In comparing these figures with those for station 3, the diurnal variation of soil temperature and the position of the stations with respect to the boundary of the irrigated area must be taken into account.

8. Potential evaporation in an irrigated area

The potential evaporation rate is usually defined as the rate of evaporation when water is available in unlimited quantity; in other words, it is the maximum possible evaporation rate under the given meteorological conditions. In an irrigated area, this maximum rate obviously depends on the distance downwind and therefore I is an unknown function of x.

When water is non-limiting, saturation will occur at the surface; hence, I must be chosen such that the resulting specific humidity at the surface equals the saturation value corresponding to the resulting surface temperature. The problem of finding $I(x)$ is solved in section A4 of the appendix on the assumption that, for the temperature range in question, the relation between saturated specific humidity and temperature is linear to a sufficient degree of approximation. This implies, of course, that the surface temperature in the irrigated area must vary in a rather narrow range.

Results of calculations of the potential evaporation rate and of surface temperature for the example of section 7 are shown in figs. 4 and 5. The evaporation rates in fig. 4 were obtained by adding $P = 0.03$ cm per day to the calculated I-values. Curve b represents the variation of the potential evaporation rate with x;

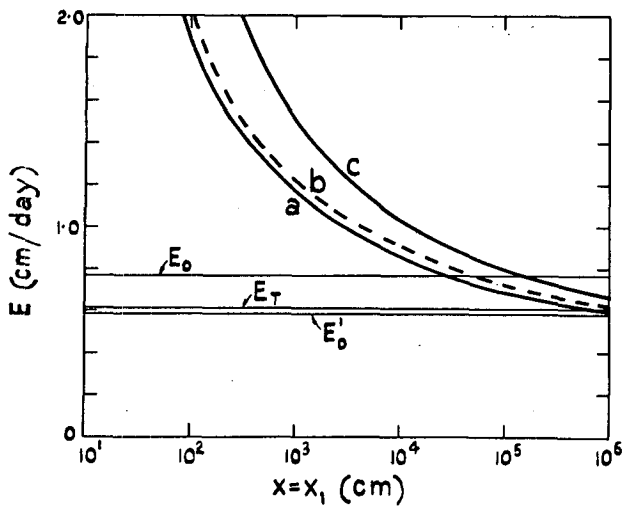

FIG. 4. Potential evaporation rates. Curve a—"lower potential evaporation rate" (*i.e.*, constant evaporation rate at which water becomes non-limiting at a fixed distance downwind, x_1) in relation to x_1; curve b—"local potential evaporation rate" in relation to distance downwind, x; curve c—"average potential evaporation rate" for an area of width x_1 in the wind direction in relation to x_1. E_0—Penman's value (calculated from dryland data) for a water surface; $E_T = 0.8\ E_0$—Penman's value for a transpiring crop; E_0'—value for a hypothetical water surface with reflectivity 0.23.

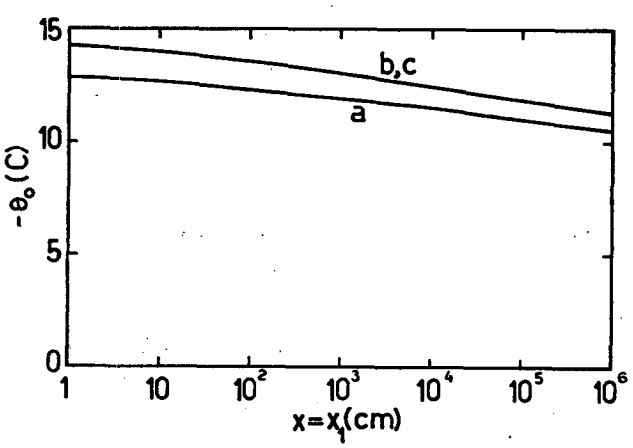

FIG. 5. Surface temperature difference between irrigated and dry land, θ_0, in relation to distance downwind. Curve a—value x_1 for the "lower potential evaporation rate"; curve b, c—values for "local and average potential evaporation rates" in relation to x.

we shall call this rate the "local potential evaporation rate," $E(x)$. Curve c shows the "average potential evaporation rate," $\bar{E}(x_1)$, for a strip of width x_1, *viz.*

$$\bar{E}(x_1) = \frac{1}{x_1}\int_0^{x_1} E(x)dx.$$

The appertaining values of θ_0 are plotted as curve b, c in fig. 5. It will be noted that the variation of surface temperature is very slight, even over a distance of 10 km. It changes from 17.1C at $x = 0$ to 20.1C at $x = 10^6$ cm. The corresponding value of s (see section A4) is 1.22×10^3C.

In practice, irrigation rates are not varied with distance downwind, at least not continuously. We have therefore also investigated the following problem: how great is the (constant) irrigation rate, I, at which water is non-limiting beyond a certain distance x_1 downwind?

Now I must be chosen such that saturation occurs at x_1. The corresponding evaporation rate is then given by $I(x_1) + P$. We shall call this rate the "lower potential evaporation rate" for an irrigated area of extent $X = x_1$ in the wind direction.

Since, for a constant I-value, θ and χ at the surface are already known as functions of I and x, it is a simple matter to find I for a given value of x_1. Results for the example of the previous section are shown as curve a in fig. 4. The corresponding values of $\theta_0(x_1)$ at the point where saturation occurs are given by curve a in fig. 5. It will be noted that the "lower potential evaporation rate" is only slightly less than the "local potential evaporation rate." This is also reflected in the small difference between curves a and b, c in fig. 5.

The present example illustrates how the potential evaporation rate can be calculated from meteorological data for the dry land. It depends, *inter alia*, on the "dryness" of the dry land (*i.e.*, on the difference between surface temperature and dew point) and on wind velocity.

It is interesting to compare the present potential evaporation rates with that calculated from the dryland data without taking advective energy and modification of the climate by irrigation into account—*e.g.*, by application of Penman's (1948) method[6]. In fig. 4, we have plotted Penman's values for E_0 and E_T. E_0 applies to a (hypothetical) water surface which absorbs 95 per cent of the incoming solar radiation and has no heat exchange with deeper layers. E_T applies to a transpiring crop; it's value was obtained by multiplying E_0 with Penman's empirical factor 0.8 for summer months. To illustrate the influence of albedo, we have calculated E_0', which applies to a hypothetical wet surface with $r = 0.23$ (the same as for the grass cover in our example) and again with no heat exchange with layers below the surface. It will be noted that, in the example, differences between the present potential evaporation rates and E_0' are appreciable up to distances of the order of 1 km. E_T is seen to be a reasonable estimate for a distance of about 10 km.

According to Penman and Schofield (1951), the difference between E_0 and E_T is for the greater part due to the difference in reflection coefficient (Penman gives $r = 0.20$ for his experiments on short grass) and for the remaining part to biological factors. In our calculations of the potential evaporation rate, the latter are not taken into account. However, their

[6] It is recognized that Penman does not consider his method to be applicable under arid or semi-arid conditions.

influence is considered to be small for a grass cover with ample water supply (Makkink, 1955, 1957; and Businger, 1956).

It follows from fig. 4 that from a viewpoint of water economy it is advantageous to have few large irrigated areas instead of many scattered small ones. This applies notably to conditions in the Australian Riverina, where irrigated pastures often have linear dimensions of the order of 100 m.

9. The horizontal and vertical extent of the influence of irrigation

We shall now briefly discuss how far the effect of an irrigated area extends in a horizontal and a vertical direction. The horizontal effect can be deduced from equation (25) of section 6.

The θ-value for a point at a distance αX to the leeward of the irrigated area is

$$\theta = F[(1 + \alpha)X] - F(\alpha X), \qquad (35)$$

and a similar equation holds for χ. When θ and χ are plotted against $\log x$ as in fig. 2, the right-hand side of (35) is given by the difference of the ordinates corresponding to two points which are $\log(1 + \alpha)/\alpha$ apart on the scale of abscissae. It will be noted that θ and χ become small when α is of the order of unity or greater. In other words, the horizontal influence of an irrigated area to its leeward extends over a distance of the order of magnitude of the area's width in the wind direction. This conclusion is confirmed by experimental findings of Dzerdzeevskii (1954).

It follows from equations (A14) and (A16) of the appendix that, when ξ is sufficiently great, θ and χ become very small if

$$(1 + z/z_1)^{1+m} \approx (1 + m)^2\xi/1.781. \qquad (36)$$

For the example of section 7, this leads to

$$z \approx 0.05x^{8/9}, \quad \text{for} \quad x \geq 10^3.$$

Hence, in this case, the influence of irrigation on air temperatures and humidities extends to a height of about $0.01x$.

10. Discussion of other work

As has been mentioned already in the introduction, much experimental work on climatic differences between irrigated and non-irrigated areas was carried out in Russia. A number of references were given, but these by no means constitute a full bibliography of the Russian work. Difficulties in obtaining papers published as symposia reports and limited translation facilities have impeded us from covering the Russian literature more completely. However, it is believed that the papers referred to here form a representative sample.

Although some of the Russian investigations were carried out in considerable detail, with measurements of temperature and humidity profiles both in vertical and horizontal directions, none of it could be analysed quantitatively because one or more of the parameters occurring in the theory were unknown. A major difference between the Russian work and our own is that the former was usually performed on isolated days, whereas our observations extended continuously over a long period. Moreover, the Russian investigations mostly refer to tall vegetation, such as wheat, cotton or orchards.

The orders of magnitude of the differences in radiation, temperature, and humidity between irrigated and non-irrigated areas, and their variation with distance downwind as observed by Russian workers agree in most cases with expectations on the basis of our experimental and theoretical results. This applies, for instance, to differences in radiation balance reported by Chudnowskii (1953) and Dzerdzeevskii (1954), to differences in temperature and humidity given by these authors, Fel'dman (1953) and Gal'tsov (1953), and to the influence found to the leeward of the irrigated area as measured by Gal'tsov (1953) and Dzerdzeevskii (1954).

Only one instance of direct measurements of potential evaporation from an irrigated field was found in the literature—*viz.*, those of Haude (1957). He determined the evaporation from bare soil in Egypt by weighing buried tanks in the middle of a 1500-m^2 wet field. Evaporation rates during the summer months were approximately 1 cm per day. This compares favourably with values expected for summer conditions at approximately the same latitude in Australia (fig. 4).

Lemon *et al* (1957) report the influence of advective energy at a distance of 10 mi downwind in an irrigated cotton field.

11. Limitations of the theory

The analysis presented in this paper strictly applies only to the case of forced convection over surfaces of uniform roughness. However, although the form of equations (16) and (21) for K_H and K_W is that for forced convection, one still is free to select the parameters K_{H0}, K_{W0} and z_1 in such a way that K_H and K_W conform as closely as possible to actual conditions. Hence, K_{H0} need not be taken equal to K_{W0}, and z_1 may differ from the roughness height.

A more serious limitation lies in the fact that diffusivities are assumed to be the same over the dry land and the irrigated area. The influence of this assumption is difficult to assess analytically; our results indicate that its effect is small, except probably with light winds, but then the interaction between dry

and irrigated areas is small anyhow. Moreover, it must be kept in mind that the present unsatisfactory state of knowledge regarding eddy diffusivities seriously limits the possibilities for further quantitative refinements.

The effect of differences in surface roughness between irrigated and non-irrigated pastures is probably small in comparison with the influence of other simplifications. The same will apply in the case of other short and dense crops. With taller dense crops, such as cereals, the influence of the vegetation on the wind profile and diffusivities is not negligible (Rider, 1954; Halstead and Covey, 1957). Possibly the present theory can be applied in a modified form by the introduction of a suitable zero-plane displacement for height in the irrigated area.

The situation is more complicated with a tall and open vegetation, as is found in orchards and vineyards. Here we have, apart from the "oasis effect," to deal with the "clothes line effect" (the term is due to Tanner, 1957) which means the effect of relatively warm and dry air blowing through the vegetation.

The theory also does not apply to evaporation from a body of water as, for example, from reservoirs and lakes where the energy conditions are quite different from those considered here due to the penetration of solar radiation to deeper layers.

Within the above-mentioned limitations, the present theory provides, apparently for the first time, a quantitative physical analysis of the influence of irrigation on the microclimate and the energy balance and a method of calculating the water needs of irrigated crops.

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MATHEMATICAL APPENDIX

A1. Derivation of solutions for θ and χ

Equations (12), (13), and (14) rewritten in dimensionless form (except for θ) are

$$\varphi \frac{\partial \theta}{\partial \xi} = \frac{\partial}{\partial \eta}\left(\psi \frac{\partial \theta}{\partial \eta}\right), \tag{A1}$$

$$-(\partial\theta/\partial\eta)_{\eta=1} = -a - B\theta_0, \tag{A2}$$

$$\theta(\xi, \infty) = 0, \tag{A3}$$

with

$$\eta = 1 + z/z_1, \quad \xi = \xi_H, \quad \varphi = u/u_0, \text{ and } \psi = K_H/K_{H0}.$$

Solutions can be found by the Laplace-transformation method (Carslaw-Jaeger, 1948). Let $\vartheta(\eta)$ be the Laplace transform of θ, then we have

$$\varphi p \vartheta = \frac{d}{d\eta}\left(\psi \frac{d\vartheta}{d\eta}\right), \tag{A4}$$

$$(-d\vartheta/d\eta)_{\eta=1} = -a/p - B\vartheta(1), \tag{A5}$$

$$\vartheta(\infty) = 0. \tag{A6}$$

For $\varphi = \eta^m$ and $\psi = \eta^m$, equation (A4) is of the Bessel type and the solution satisfying (A6) is

$$\vartheta \sim \eta^{(1-n)/2} H^{(1)}_{\pm\frac{1-n}{2-n+m}}\left(i\frac{2p^{\frac{1}{2}}\eta^{(2+m-n)/2}}{2+m-n}\right), \tag{A7}$$

where H is a Bessel function of the third kind. The proportionality factor follows from (A5).

Solutions in the form of tabulated functions are obtained in the following cases.[7]

(a) $n = m = 0$—*i.e.*, eddy conductivity and wind velocity constant with height.

The solutions (Carslaw-Jaeger, 1948) are

$$\theta = -\frac{a}{B}\left\{\operatorname{erfc}\frac{\eta-1}{2\xi^{\frac{1}{2}}} - \exp\left[B(\eta-1) + B^2\xi\right] \times \operatorname{erfc}\left(\frac{\eta-1}{2\xi^{\frac{1}{2}}} + B\xi^{\frac{1}{2}}\right)\right\}, \tag{A8}$$

$$\chi = D\{2\pi^{-\frac{1}{2}}\xi^{\frac{1}{2}}\exp[-(\eta-1)^2/4\xi] - (\eta-1)\operatorname{erfc}[(\eta-1)/2\xi^{\frac{1}{2}}]\}. \tag{A9}$$

(b) $n = \frac{1}{3}(4+m)$, m arbitrary positive.

[7] The mathematics for the transformed variable is analogous to that for periodic solutions (de Vries, 1957).

From (A7) and (A5) we obtain

$$\vartheta = -\frac{a\eta^{-(1+m)/3}\exp[-3p^{\frac{1}{2}}(1+m)^{-1}(\eta^{(1+m)/3}-1)]}{[B+\frac{1}{3}(1+m)+p^{\frac{1}{2}}]p} \tag{A10}$$

and

$$\theta = -\frac{a\eta^{-(1+m)/3}}{B+\frac{1}{3}(1+m)}\left\{\operatorname{erfc}\frac{3(\eta^{(1+m)/3}-1)}{2(1+m)\xi^{\frac{1}{2}}} - \exp\left[\frac{3(B+\frac{1}{3}+\frac{1}{3}m)(\eta^{(1+m)/3}-1)}{1+m} + (B+\tfrac{1}{3}+\tfrac{1}{3}m)^2\xi\right] \times \operatorname{erfc}\left[\frac{3(\eta^{(1+m)/3}-1)}{2(1+m)\xi^{\frac{1}{2}}} + (B+\tfrac{1}{3}+\tfrac{1}{3}m)\xi^{\frac{1}{2}}\right]\right\}. \tag{A11}$$

The corresponding solution for χ is found by substituting $-a = D$ and $B = 0$.

(c) $n = 1$, m arbitrary positive.

The solution for ϑ now becomes

$$\vartheta = \frac{-aK_0[2p^{\frac{1}{2}}(1+m)^{-1}\eta^{(1+m)/2}]}{p\{p^{\frac{1}{2}}K_1[2p^{\frac{1}{2}}(1+m)^{-1}] + BK_0[2p^{\frac{1}{2}}(1+m)^{-1}]\}}, \tag{A12}$$

from which (17) is found by the usual procedure. Equation (22) follows from (17) by the substitutions $-a = D$ and $\beta = 0$.

The cases treated here by no means exhaust the possibilities of finding solutions to (A1), (A2), and (A3), of course. However, no other manageable solutions that approximate actual conditions more accurately than the present ones were found. The solutions under (b) and (c) differ from actual conditions mainly in two respects: (i) the infinite increase of u and K with height, the influence of which is discussed in section A3; (ii) the non-zero value of φ for $z = 0$. The influence of the latter factor is restricted to a shallow layer of air (with thickness of the order z_1) and is considered to be of little importance.

In applications to actual data, case (c) has been used. Cases (a) and (b) provide convenient additional solutions for rapid exploratory calculations.

A2. Asymptotic solutions

Asymptotic expansions for large ξ of the solutions (A8), (A9), and (A11) follow from the well-known asymptotic expansions of erfc and the exponential function.

The method of obtaining aymptotic expansions in case (c) has been discussed in detail by Ritchie and Sakakura (1956). Expanding the Bessel functions in (A12) after powers of p we obtain

$$\vartheta = \frac{a[\ln C^2\rho + \rho\ln C^2\rho - 2\rho + 0(p^2\ln p)]}{(1+m)p[1+\rho_0\ln C^2\rho_0 - \rho_0 - \frac{1}{2}\beta(\ln C^2\rho_0 + \rho_0\ln C^2\rho_0 - 2\rho_0) + 0(p^2\ln p)]}, \tag{A13}$$

with $\rho = p(1+m)^{-2}\eta^{1+m}$, $\rho_0 = p(1+m)^{-2}$ and $C = e^{\gamma} = 1.781$.

From (A13), we have derived two different asymptotic expressions for θ, one for small $\beta \ln p$, the other for large $\beta \ln p$. The first expression is[8]

$$\theta = -(a/B)\{\tfrac{1}{24}\beta^2\pi^2 + \tfrac{1}{4}\beta^3\zeta(3) - B\ln\eta + (1 + B\ln\eta)[\tfrac{1}{2}\beta(1 - \tfrac{1}{8}\beta^2\pi^2)\ln\xi' - \tfrac{1}{4}\beta^2\ln^2\xi' + \tfrac{1}{8}\beta^3\ln^3\xi' + 0(\beta^4\ln^4\xi')] + 0[\beta\xi^{-1}(\ln\xi + \tfrac{1}{2}\eta^{1+m})]\}, \quad \text{(A14)}$$

with $\xi' = (1+m)^2\xi/C$ and $\zeta(3) = 1.202$ (ζ is the Riemann zeta-function). This expression can be applied when

$$(\tfrac{1}{2}\beta\ln\xi')^3 \ll 1 \quad \text{and} \quad \eta^{1+m}/(\xi\ln\xi) \ll 1.$$

When $\frac{1}{2}\beta\ln\xi' \ll 1$, we find

$$\theta = -(a/B)\{1 - (1 + B\ln\eta)[(\tfrac{1}{2}\beta\ln\xi')^{-1} - (1 + \tfrac{1}{2}\gamma\beta)(\tfrac{1}{2}\beta\ln\xi')^{-2} + 0(\tfrac{1}{2}\beta\ln\xi')^{-3}]\}. \quad \text{(A15)}$$

The asymptotic solution for χ becomes

$$\chi = \frac{D}{1+m}\left\{-(1+m)\ln\eta + \ln\xi' + \frac{2\ln\xi'}{(1+m)^2\xi} + \frac{1 + \eta^{1+m} - (1+m)\ln\eta}{(1+m)^2\xi} + 0(\xi^{-2})\right\}. \quad \text{(A16)}$$

A3. Asymptotic solutions for bounded K

It has already been mentioned that at great heights (say $z > 100$m) equation (15) and (16) no longer represent actual conditions to a reasonable degree of approximation. For small values of x, this fact will be of little consequence, but one must inquire at what distance downwind the formulae of the preceding sections are no longer applicable. In order to investigate this question, asymptotic solutions have been developed for bounded u and K.

In the case studied, u and K are considered to be constant for $z > Z$—*i.e.*, they are given by (15) and (16) for $z \leq Z$, whilst for $z > Z$ we have

$$u = u_0(1 + Z/z_1)^m, \quad \text{(A17)}$$

$$K = K_0(1 + Z/z_1). \quad \text{(A18)}$$

The solution for ϑ can be obtained by combining the cases (a) and (c) of section A1 in a two-layer problem with conditions of continuity of temperature and heat flux at $z = Z$ (de Vries, 1957). We then find

$$\vartheta = -aN/p\Delta, \quad \text{(A19)}$$

with

$$N = [I_1(Rp^{\frac{1}{2}}) + I_0(Rp^{\frac{1}{2}})]K_0(rp^{\frac{1}{2}}) + [K_1(Rp^{\frac{1}{2}}) - K_0(Rp^{\frac{1}{2}})]I_0(rp^{\frac{1}{2}}),$$

$$\Delta = [I_1(Rp^{\frac{1}{2}}) + I_0(Rp^{\frac{1}{2}})]K_1(r_0p^{\frac{1}{2}}) - [K_1(Rp^{\frac{1}{2}}) - K_0(Rp^{\frac{1}{2}})]I_1(r_0p^{\frac{1}{2}}) + B[I_1(Rp^{\frac{1}{2}}) + I_0(Rp^{\frac{1}{2}})]K_0(r_0p^{\frac{1}{2}}) + B[K_1(Rp^{\frac{1}{2}}) - K_0(Rp^{\frac{1}{2}})]I_0(r_0p^{\frac{1}{2}}),$$

$$r = 2(1+m)^{-1}\eta^{(1+m)/2}, \quad r_0 = 2(1+m)^{-1}, \quad R = 2(1+m)^{-1}(1 + Z/z_1)^{(1+m)/2}.$$

From (A19), the following two asymptotic expansions were derived. The first one is

$$\theta = -\frac{a}{B}\left\{\frac{B[(1+\alpha_1)\ln H/\eta - \alpha_2(1+m)^{-1}]}{1 + B\ln H} + \frac{B\alpha_3}{(1+m)(1+B\ln H)H^{1+m}} + \frac{2B(1+\alpha_4)\xi^{\frac{1}{2}}}{\pi^{\frac{1}{2}}(1+B\ln H)H^{(1+m)/2}} - \frac{4B^3\xi^{\frac{3}{2}}}{3\pi^{\frac{1}{2}}(1+B\ln H)^3H^{3(1+m)/2}} + 0\left(\frac{BH^{(1+m)/2}}{\xi^{\frac{1}{2}}}\right)\right\}, \quad \text{(A20)}$$

for $B\xi^{\frac{1}{2}}/(1 + B\ln H)H^{(1+m)/2} \ll 1$ and $H^{1+m}/\xi \ll 1$, where

$$\alpha_1 = 1 + \frac{\beta(1 + B\ln H - \frac{1}{2}\beta)}{(1+B\ln H)^2} - \frac{\beta(1 - \frac{1}{2}\beta)}{(1+B\ln H)^2H^{1+m}},$$

$$\alpha_2 = 1 - \frac{3\beta}{2(1+B\ln H)} + \frac{\beta^2}{(1+B\ln H)^2},$$

$$\alpha_3 = \frac{1 - \frac{1}{2}\beta}{1+B\ln H} - \frac{\beta[1 + (1+m)\ln H - \beta]}{(1+B\ln H)^2},$$

$$\alpha_4 = \alpha_1 - \frac{B\ln H}{1+B\ln H}.$$

Equation (A20) corresponds to (A14); for $B\xi^{\frac{1}{2}}/H^{(1+m)/2} \gg 1$ we find

$$\theta = -\frac{a}{B}\left\{1 - \frac{H^{(1+m)/2}(1 + B\ln H)}{\pi^{\frac{1}{2}}B\xi^{\frac{1}{2}}} + 0\left[\frac{H^{3(1+m)/2}(1 + B\ln H)^3}{B^3\xi^{\frac{3}{2}}}\right]\right\}. \quad \text{(A21)}$$

The asymptotic solution for χ is

$$\chi = D\{\ln H/\eta - (1+m)^{-1} + (1+m)^{-1}H^{-(1+m)} + 2\pi^{-\frac{1}{2}}\xi^{\frac{1}{2}}H^{-(1+m)/2} + 0(H^{(1+m)/2}\xi^{-\frac{1}{2}})\}, \quad \text{(A22)}$$

for $H^{1+m}/\xi \ll 1$.

[8] For simplicity, we have written $\xi = \xi_H = \xi_W$ throughout the appendix; hence, $\xi = \xi_H$ in the expression for θ and $\xi = \xi_W$ in the expressions for χ.

The influence on θ and χ of the behavior of u and K for $z > Z$ can be assessed by comparing the asymptotic solutions (A20), (A21), and (A22) with the corresponding solutions of section A2. When necessary, the former can be used.

A4. Potential evaporation

When water is non-limiting, the evaporation rate is determined by atmospheric conditions. $I(x)$ is now an unknown function that must be found by making use of the condition that the air is saturated at the surface.

Let L denote the Laplace transform operator and L^{-1} the inverse operator. Then we have

$$\vartheta(p,\eta) = p\vartheta_1 L\{I(\xi)\}, \tag{A23}$$

where ϑ_1 is given by (A12) with $a_1 = a/I$ substituted for a. Using the "Faltung" theorem, we obtain

$$\theta(\xi, \eta) = \int_0^\xi I(u) f(\xi - u, \eta) du, \tag{A24}$$

with

$$f(\xi,\eta) = L^{-1}\{p\vartheta_1\}.$$

Similarly,

$$\chi(\xi, \eta) = \int_0^\xi I(u) g(\xi - u, \eta) du, \tag{A25}$$

with

$$g(\xi,\eta) = L^{-1}\{p\tau_1\},$$

where τ_1 is found from ϑ_1 by taking $B = 0$ and substituting $D_1 = D/I$ for $-a_1$.

The condition of saturation at the surface leads to a relation between $\theta(\xi,1)$ and $\chi(\xi,1)$ from which I must be solved. We shall suppose that, in the range of surface temperatures found in the irrigated area, the temperature can be represented to a sufficient degree of approximation as a linear function of the saturated specific humidity, $q_{\text{sat}}(T)$, *viz.*

$$T = sq_{\text{sat}}(T) + c. \tag{A26}$$

Then we find, from (A24), (A25), and (A26),

$$\int_0^\xi I(u)[f(\xi - u, 1) - sg(\xi - u, 1)]du = d, \tag{A27}$$

where d is the difference between temperature and dew point at the surface in the dry land. This is an integral equation of Abel's type that can be solved by the Laplace transformation method. Applying the "Faltung" theorem, we find

$$L\{I(\xi)\} = \frac{dp^{\frac{1}{2}}K_1(r_0p^{\frac{1}{2}})[p^{\frac{1}{2}}K_1(r_0p^{\frac{1}{2}})+BK_0(r_0p^{\frac{1}{2}})]}{p[(a_1+sD_1)p^{\frac{1}{2}}K_1(r_0p^{\frac{1}{2}})K_0(r_0p^{\frac{1}{2}})+sBD_1K_0^2(r_0p^{\frac{1}{2}})]}, \tag{A28}$$

from which I can be found by the usual procedure. However, the resulting expression is unwieldy and inconvenient for numerical calculation.

An asymptotic solution for I can be readily derived from (A28) with the use of the transforms listed by Ritchie and Sakakura. We find

$$\begin{aligned} I(\xi) = \frac{(1+m)d}{a_1+sD_1}\Bigg\{ & \frac{1}{\ln \xi'/C}\Bigg[1 - \frac{0.5772}{\ln \xi'/C} - \frac{1.312}{\ln^2 \xi'/C} \\ & + \frac{0.2520}{\ln^3 \xi'/C} + \frac{3.997}{\ln^4 \xi'/C} + \frac{5.064}{\ln^5 \xi'/C} + 0(\ln^{-6} \xi)\Bigg] \\ & + (\tfrac{1}{2}\beta - \beta_1) - \beta_1(\tfrac{1}{2}\beta - \beta_1) \ln \xi' \\ & + \beta_1^2(\tfrac{1}{2}\beta - \beta_1)(\ln^2 \xi' - \tfrac{1}{6}\pi^2) \\ & + 0(\beta^4 \ln^3 \xi) + 0(\xi^{-1} \ln^{-1} \xi)\Bigg\}, \end{aligned} \tag{A29}$$

with $\beta_1 = \frac{1}{2}\beta sD_1/(a_1 + sD_1)$. For small ξ-values, we have

$$\begin{aligned} I(\xi) = \frac{d}{a_1 + sD_1}[(\pi\xi)^{-\frac{1}{2}} + \tfrac{1}{4}(1 + m) \\ + B - (1 + m)\beta_1 + 0(\xi^{\frac{1}{2}})]. \end{aligned} \tag{A30}$$

We shall term I the "local potential irrigation rate"; the "average potential irrigation rate" for a strip of width x_1 in the wind direction is defined as

$$\bar{I} = \frac{1}{x_1}\int_0^{x_1} I(x)dx = \frac{1}{\xi_1}\int_0^{\xi_1} I(\xi)d\xi. \tag{A31}$$

Now we have

$$L\{\bar{I}(\xi_1)\} = p^{-1}L\{I(\xi_1)\}, \tag{A32}$$

from which the following asymptotic expression is found:

$$\begin{aligned} \bar{I}(\xi_1) = \frac{(1+m)d}{a_1+sD_1}\Bigg\{ & \frac{1}{\ln \xi_1'/C}\Bigg[1 + \frac{0.4228}{\ln \xi_1'/C} - \frac{0.4662}{\ln^3 \xi_1'/C} \\ & - \frac{1.147}{\ln^3 \xi_1'/C} - \frac{0.5892}{\ln^4 \xi_1'/C} + \frac{2.117}{\ln^5 \xi_1'/C} + 0(\ln^{-6} \xi_1)\Bigg] \\ & + (\tfrac{1}{2}\beta - \beta_1) - \beta_1(\tfrac{1}{2}\beta - \beta_1)(\ln \xi_1' - 1) \\ & + \beta_1^2(\tfrac{1}{2}\beta - \beta_1)(\ln^2 \xi_1' - 2 \ln \xi_1' + 2 - \tfrac{1}{6}\pi^2) \\ & + 0(\beta^4 \ln^3 \xi_1) + 0(\xi_1^{-1} \ln^{-1} \xi_1)\Bigg\}. \end{aligned} \tag{A33}$$

From (A23) and (A28), we find

$$\vartheta(p, 1) = -\frac{a_1dp^{\frac{1}{2}}K_1(r_0p^{\frac{1}{2}})K_0(r_0p^{\frac{1}{2}})}{p[(a_1+sD_1)p^{\frac{1}{2}}K_1(r_0p^{\frac{1}{2}})K_0(r_0p^{\frac{1}{2}})+sBD_1K_0^2(r_0p^{\frac{1}{2}})]}, \tag{A34}$$

from which the following expressions for the surface temperature are derived:

$$\theta_0 = -\frac{a_1 d}{a_1 + sD_1}\{1 - \beta_1 \ln \xi' + \beta_1^2(\ln^2 \xi' - \tfrac{1}{6}\pi^2) - \beta_1^3[\ln^3 \xi' - \tfrac{1}{2}\pi^2 \ln \xi' + 2\zeta(3)] + 0(\beta^4 \ln^4 \xi) + 0(\beta\xi^{-1} \ln \xi)\}, \quad \text{(A35)}$$

$$\theta_0 = -\frac{a_1 d}{a_1 + sD_1}[1 - 2\pi^{-\frac{1}{2}}(1+m)\beta_1 \xi^{\frac{1}{2}} + 0(\xi)]. \quad \text{(A36)}$$

In actual calculations, one first determines the range of surface temperature approximately by using a tentative value of s in (A35). A more accurate s-value is then easily obtained by trial and error.

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